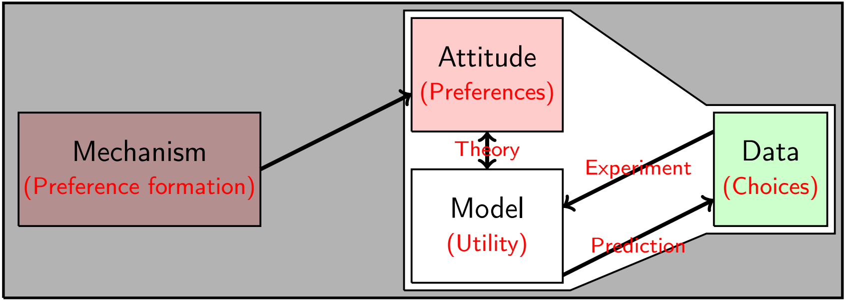
Choice

Theory of Individual and Strategic Decisions

MSc Human Decision Science

Maastricht University

- This week we will focus on testing models using choice data.
- This methodology is called **revealed preference**



- A choice problem is the set of available alternatives in some instance $A \subseteq X$.
- We often call it a **menu**
- An experiment is a set of menus from which we eventually observe choices

$$\mathcal{E} = \{A_1, A_2, \ldots, A_n\}$$

- If the experiment contains all nonempty menus, it is called complete experiment
- The data is described with a **choice function**,

$$c(A) \in A \text{ is the choice made from menu } A \subseteq X$$

Example

The alternatives are $X = \{\text{apple } (a),\ \text{banana } (b),\ \text{carrot } (c)\}$.
Each menu is a grocery store that sells some of these fruits:

$$\begin{aligned} \text{Albert Heijn } (A_1) &= \{a, b, c\} \\ \text{Jumbo } (A_2) &= \{a, b\} \\ \text{Jan Linders } (A_3) &= \{b, c\} \\ \text{Plus } (A_4) &= \{a, c\} \end{aligned}$$

The choice function reveals what the DM chooses from each store:

$$\begin{aligned} c(A_1) &= a \\ c(A_2) &= a \\ c(A_3) &= b \\ c(A_4) &= a \end{aligned}$$

If we observe purchases from each of these stores, then we have a complete experiment.

- To make sense of the observed choices, we need to assume that they are somehow logically consistent

Assumption

*The DM acts as if he was **rational**:*

1. *The DM has a preference relation (i.e., utility function exists)*
2. *When he faces a problem set, he is aware of the available choices*
3. *He chooses the most preferred available choice (i.e., he maximizes utility)*

- What does "as if" mean?
- Note that "rational choice" does not necessarily mean "best choice".

- We want to empirically test the previous assumption, i.e.,

$$H_0 : \textit{the individual chooses as if he was rational.}$$

- Mathematically, we ask:

Is there a $u : X \to \mathbb{R}$ *such that* $u(c(A)) \geq u(a)$ *for all* $a \in A$*?*

or equivalently

Is there some $\succeq$ *such that* $c(A) \succeq a$ *for all* $a \in A$*?*

- We will need some criterion to test the hypothesis.

criterion passes	:	choice function is **rationalizable**
criterion fails	:	choice function is **not rationalizable**

- Notice that we do not say "rational" but "rationalizable (by some preference relation)"
  - We can never test if he is actually rational (preferences are unobservable)
  - We can only test if he behaves as if he was rational (only choices are observable)

Proposition

Every choice function is rationalizable (i.e., H_0 is non-falsifiable).

Example

The choice function:

$$
\begin{aligned}
c(\{a,b,c\}) &= a \\
c(\{a,b\}) &= a \\
c(\{b,c\}) &= b \\
c(\{a,c\}) &= c
\end{aligned}
$$

It is rationalizable by u if the following happens:

$$
\begin{aligned}
u(a) &\geq u(b), u(c) \\
u(a) &\geq u(b) \\
u(b) &\geq u(c) \\
u(c) &\geq u(a)
\end{aligned}
$$

So the following utility function works: $u(a) = u(b) = u(c) = 1$

- We impose a stronger assumption:

Assumption

The DM acts as if he was ***rational*** *and there are no indifferences:*

1. *The DM has an antisymmetric preference relation*
2. *When he faces a problem set, he is aware of the available choices*
3. *He chooses the most preferred available choice*

- We instead test the following hypothesis:

$$H_1 : \textit{the individual chooses as if he was rational and there are no indifferences}$$

- Mathematically, we ask:

Is there some $u : X \to \mathbb{R}$ assigning a different utility to each alternative, such that $u(c(A)) > u(a)$ for all $a \in A$?

or equivalently

Is there some antisymmetric $\succeq$ such that $c(A) \succ a$ for all $a \in A$?

Definition

A choice function c is said to satisfy property α if for any two choice problems $A, B \subseteq X$ such that $B \subset A$, the following holds:

$$\text{if } c(A) \in B \text{ then } c(B) = c(A)$$

- What does this say?
- Property α can be checked in a dataset, even if the experiment $\mathcal{E}$ is not complete.

Proposition (Proposition 2.1 in the textbook)

If property α is violated, c is not rationalized (by any antisymmetric $\succeq$), i.e., H_1 is rejected.

- Any ideas why?
- Notice that this is true even if the experiment is not complete.

Definition

A choice function c is said to satisfy property α if for any two choice problems $A, B \subseteq X$ such that $B \subset A$, the following holds:

$$\text{if } c(A) \in B \text{ then } c(B) = c(A)$$

- What does this say?
- Property α can be checked in a dataset, even if the experiment $\mathcal{E}$ is not complete.

Proposition (Proposition 2.2 in the textbook)

If property α is satisfied and $\mathcal{E}$ is complete, c is rationalized (by some antisymmetric $\succeq$), i.e., H_1 is not rejected.

- Any ideas why?
- Notice that this is true only if the experiment is complete.

Example

Consider the following dataset:

$$
\begin{aligned}
c(\{a, b\}) &= a \\
c(\{b, c\}) &= b \\
c(\{a, c\}) &= c
\end{aligned}
$$

- Two questions:
  1. Does this satisfy property α?
  2. Is this data rationalized by any antisymmetric preference $\succeq$?
- Is there a discrepancy
- Use this example in question 20 on Thursday:

- Two main problems exist here
    1. We assume antisymmetric preferences (too restrictive models)
    2. We need complete experiments (too much data)
- We will revisit these problems now.
- What follows is very standard in the literature and in practice, but it is not in the book.

- The data is described with a **choice correspondence**,

  $C(A) \subseteq A$ are the choices made from menu $A \subseteq X$

Example

The alternatives are still $X = \{\text{apple } (a),\ \text{banana } (b),\ \text{carrot } (c)\}$:

$$\begin{aligned}
\text{Albert Heijn } (A_1) &= \{a, b, c\} \\
\text{Jumbo } (A_2) &= \{a, b\} \\
\text{Jan Linders } (A_3) &= \{b, c\} \\
\text{Plus } (A_4) &= \{a, c\}
\end{aligned}$$

The choice correspondence reveals what the DM chooses from each store:

$$\begin{aligned}
C(A_1) &= \{a, b\} \\
C(A_2) &= \{a\} \\
C(A_3) &= \{c\} \\
C(A_4) &= \{a, c\}
\end{aligned}$$

Definition

We say that a is **revealed preferred** to b (by C) if

$$\text{there is some } A \text{ such that } a, b \in A \text{ and } a \in C(A).$$

Then we write $a \succeq_C b$.

Definition

We say that a is **strictly revealed preferred** to b (by C) if

$$\text{there is some } A \text{ such that } a, b \in A \text{ and } a \in C(A) \text{ and } b \notin C(A).$$

Then we write $a \succ_C b$.

Definition

The **Generalized Axiom of Revealed Preference (GARP)** is violated if

$$\text{there is some sequence } a_1, \ldots, a_n \text{ such that}$$
$$a_1 \succeq_C \cdots \succeq_C a_n \succ_C a_1.$$

Theorem

C is rationalized (by some $\succeq$) if and only if it satisfies GARP.

+ The preference relation does not need to be antisymmetric.
+ The experiment does not need to be complete.
− Difficult to collect choice correspondence

Example

The data that we observe are:

$$C(\{a,b\}) = \{a,b\}$$
$$C(\{b,c\}) = \{b\}$$
$$C(\{a,c\}) = \{c\}$$

The implication is

$$a \succeq_C b \succ_C c \succ_C a$$

meaning that GARP is violated: No utility function that would lead to these choices.

- We only tested the most basic rationality assumption(s), i.e.,

  Is there a utility function consistent with the observed choices?

- We can have stricter tests by adding more assumptions, e.g.,

  Is there a utility function of the form $u(x) = \ldots$ which is consistent with the observed choices?

- Usually we will do this when the context (i.e., the choice domain X) is specified

  Next week the choice domain will be the set of all lotteries (in order to study uncertainty), and the models that we will study will be expected utility.

- Revealed preference tests are different from statistical tests:

previous tests	:	one violation	$\Rightarrow$	immediate rejection
statistics	:	one violation	$\Rightarrow$	higher probability of rejection

- There are also tests based on **Rationality index**:

  What proportion of the dataset to I need to drop in order to pass GARP?

- At home you need to read Chapter 2 of the textbook.
- Diagonal reading is not enough.
- You need to read carefully until you understand the line of thinking.
- You will receive a sheet with questions that you need to prepare before the tutorial.
- Avoid using ChatGPT before the tutorial (in any case you are not graded). If you decide to use it, you are asked to report it.